Translating Bimodal Size Spectra to Gamma Functions And Inferring Ice Particle Mass-Dimension Relationships

David L. Mitchell
Desert Research Institute, Reno, Nevada

Brad Baker, Paul Lawson, Bryan Pilson and Qixu Mo
SPEC, Inc., Boulder, Colorado

Motivation for Study

1. Many cloud resolving models represent particle size distributions (PSD) as gamma functions. Cloud radiative properties can also be calculated from the PSD if they are represented as gamma functions (e.g. MADA). Thus an accurate means of representing PSDs as gamma functions is needed.

2. Power-law mass-dimension expressions for ice particles are used in many treatments of ice cloud microphysics. In spite of recent attempts to determine representative expressions for the PSD, an analytical means of quantifying both constants α and β has not been found (where $m = \alpha D^{\beta}$).

3. Ice cloud sedimentation rates and radiative properties depend on the ice particle area- and mass-dimension relationships.

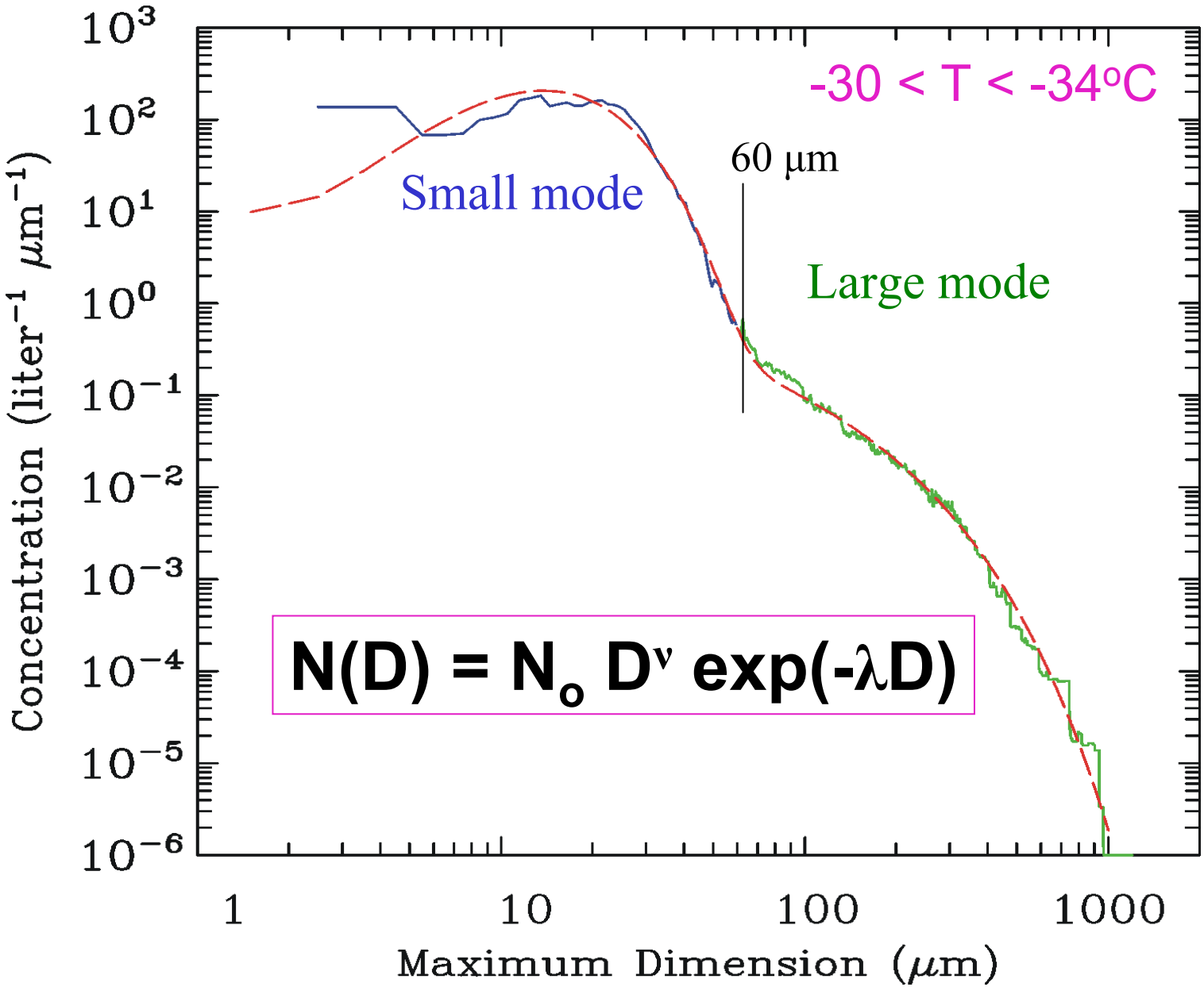
Ice Particle Size Distributions (PSD)

- PSD from Lawson et al. (2006): In Situ observations of the microphysical properties of wave, cirrus and anvil clouds. Part II: Cirrus Clouds. *J. of Atmos. Sci.*, In Press. Available at: www.specinc.com/publications.htm

- Mid-latitude (non-convective) cirrus PSD obtained during 22 Learjet flight missions, 104 horizontal legs (1 PSD per leg) and over 15,000 km of in-cloud sampling, $-28C \leq T \leq -61C$

- Measured by:

  FSSP for 2 μm < D < 30 μm,
  CPI for 30 μm < D < 200 μm, and
  2DC for 200 μm < D < 2000 μm


-30 < T < -34°C
60 μm
Small mode
Large mode
N(D) = N_o D^ν exp(-λD)
Concentration (liter^-1 μm^-1)
Maximum Dimension (μm)
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
1
10
100
1000

Methodology

Calculation of dispersion parameter ν for small particle mode:

$$\nu = \frac{(\beta + 0.67)\ \bar{D} - D_m}{D_m - \bar{D}}$$

And large particle mode:

$$\nu = \frac{(2\beta + 0.67)\ \bar{D} - D_Z}{D_Z - \bar{D}}$$

where

ice particle mass = m = αD^{β}

Calculation of $\bar{D}$, D_m and D_Z

$$\bar{\mathbf{D}} = \sum_{i=1}^{n} D_i N_i / \sum N_i$$

$$Z/2 = \sum_{i=1}^{n(Dz)} (\alpha D_i^{\beta})^2 N_i$$

$$IWC/2 = \sum_{i=1}^{n(Dm)} \alpha D_i^{\beta} N_i$$

$$Z = \sum_{i=1}^{n} (\alpha D_i^{\beta})^2 N_i$$

$$IWC = \sum_{i=1}^{n} \alpha D_i^{\beta} N_i$$

Other Gamma Parameters

$$\lambda = (v + 1) / \bar{D}$$

$$N_o = \frac{N \lambda^{v+1}}{\Gamma(v + 1)}$$ (for small mode)

$$N_o = \frac{N(D)}{D^v \exp(-\lambda D)}$$ (for large mode)

Technique

1. Use the entire measured PSD when calculating ν and λ for the large mode. This gives a negative value for ν (super-exponential) and insures good continuity with the large mode.

2. The N_o for the large mode can be evaluated at the large mode median mass dimension, D_m.

3. Use m-D expressions corresponding to quasi-spherical and/or compact ice particles for the small mode.

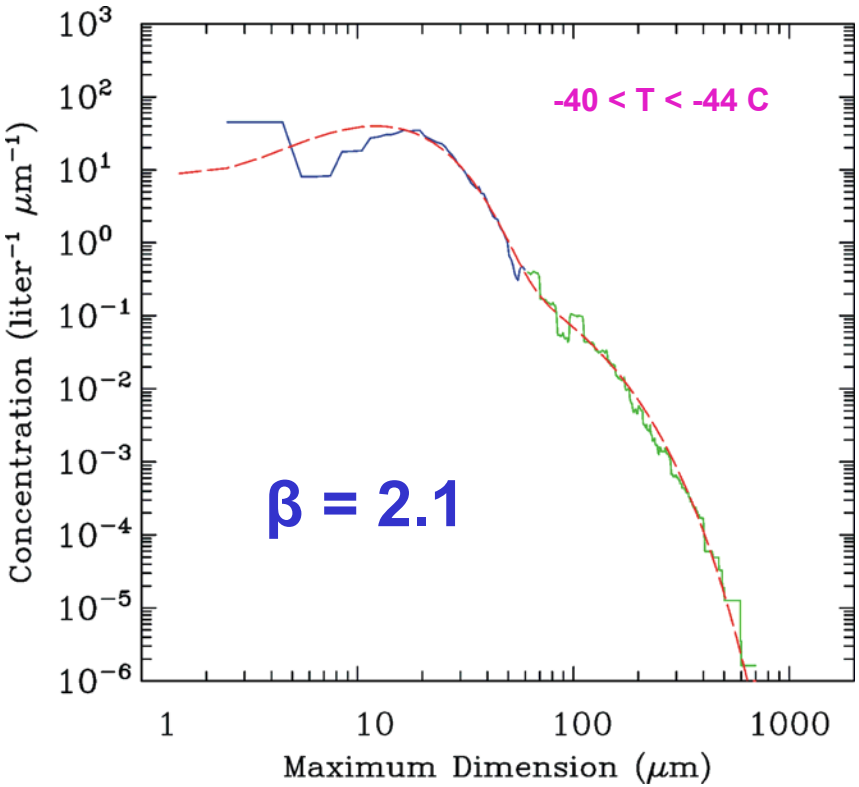
Ice Particle Mass Expressions Representative for the Large Mode PSD

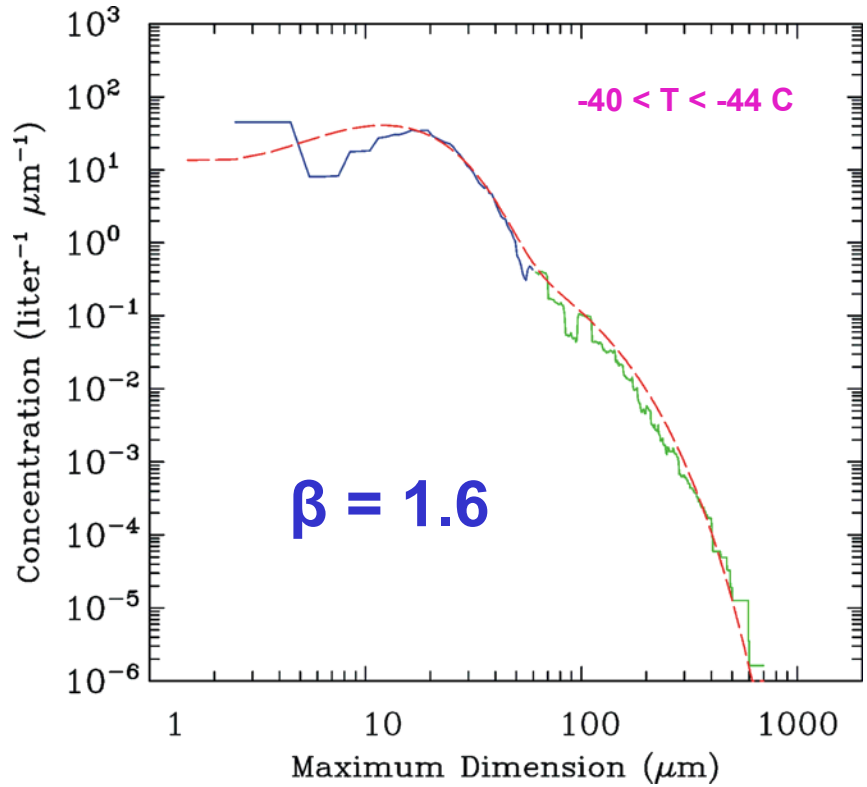
The dependence of the PSD on β can be inferred by differentiating the ice particle mass expression: $m = \alpha D^{\beta}$

$$\frac{dD}{dt} = (\alpha\beta)^{-1} D^{1-\beta} \frac{dm}{dt}$$

In practice, β is determined for the large mode by optimizing the large mode gamma-fit with the observed PSD. Note ν depends on β.

Sensitivity of large mode PSD fit to β

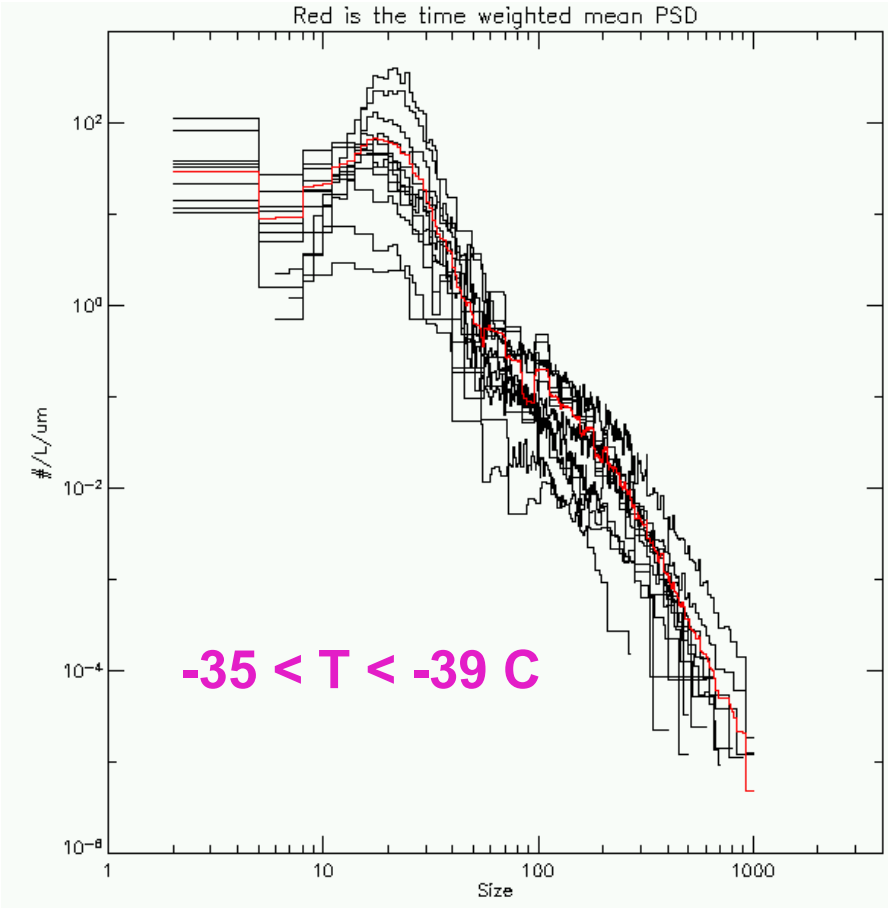


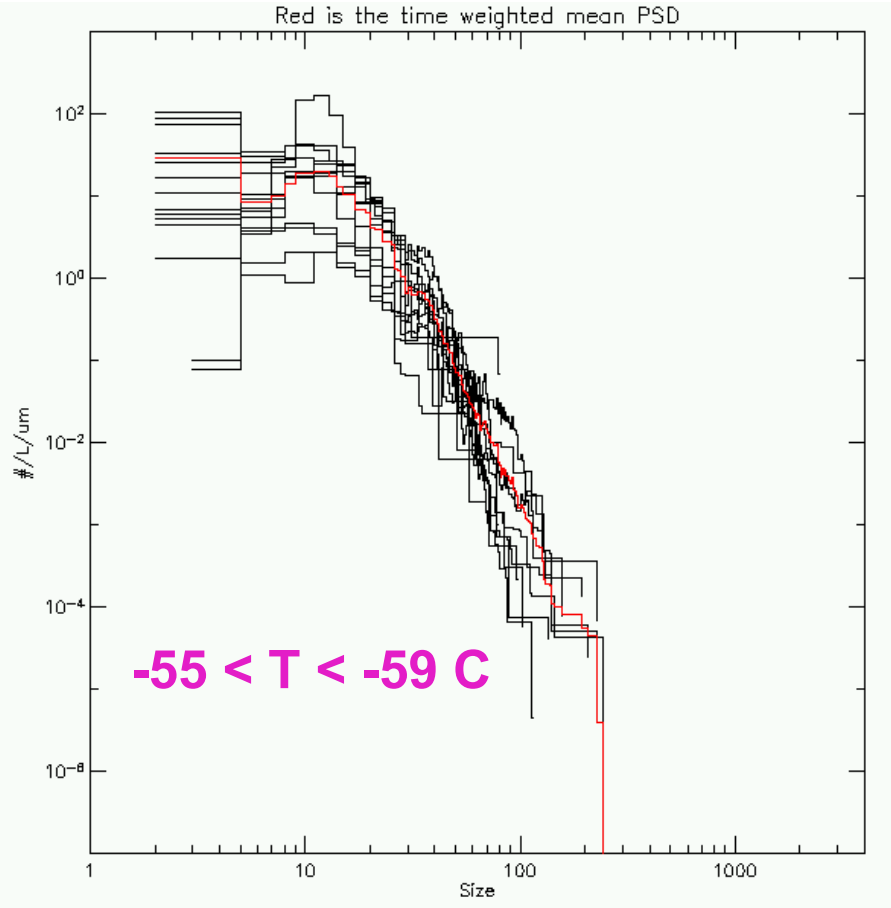


To calculate α, an independent measurement of the PSD IWC is needed (e.g. CVI or Nevzorov probe). Since the m-D relationship for the small mode is not too uncertain, then using the estimated β for the large mode, α_2 is incremented from low values until the measured and calculated IWC are equal:

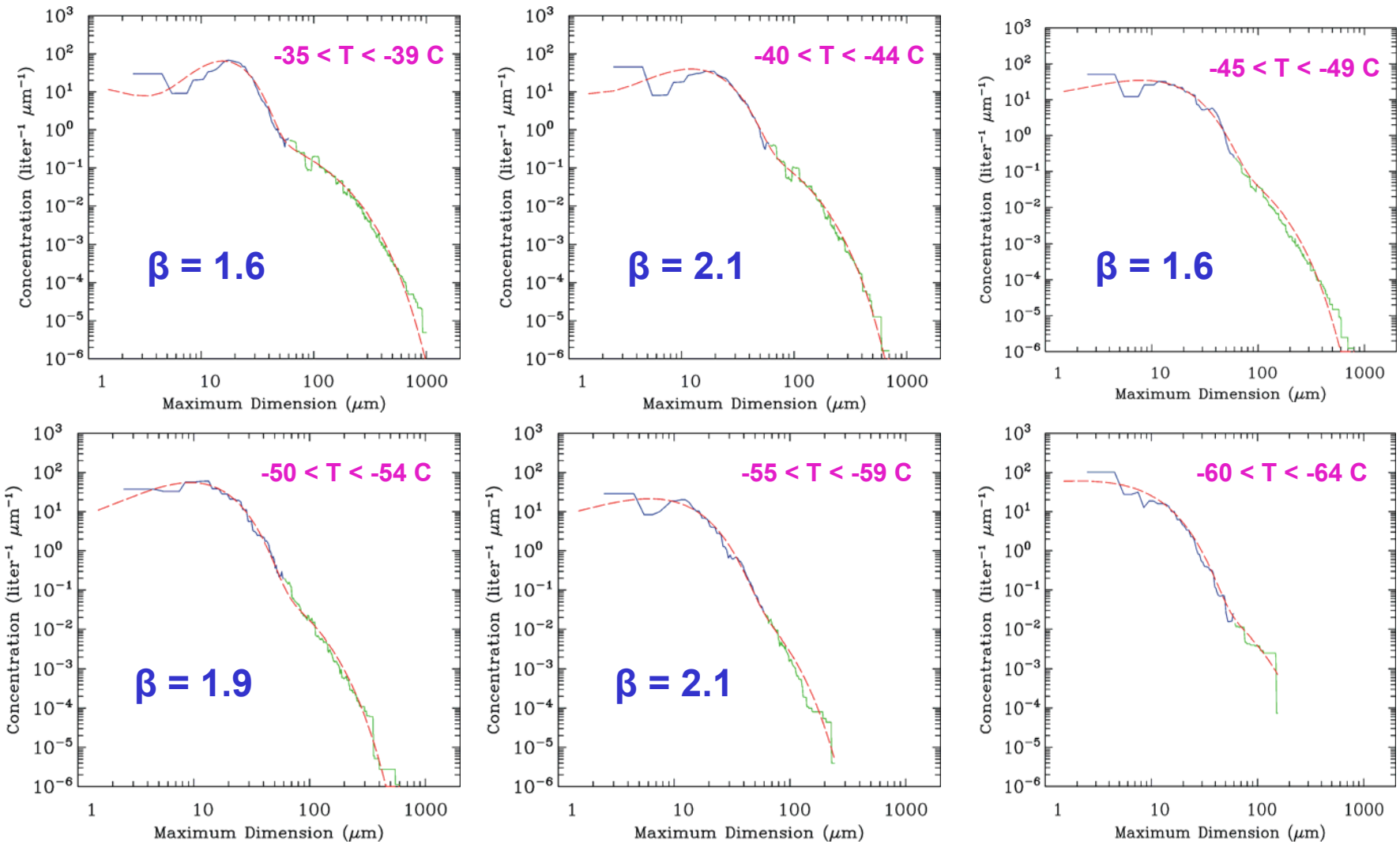
$$IWC = \int_{0}^{D=60\ \mu m} \alpha_1 D^{\beta} N(D)dD + \int_{D=60\ \mu m}^{Dmax} \alpha_2 D^{\beta} N(D)dD$$

Natural PSD Variability within Temperature Intervals





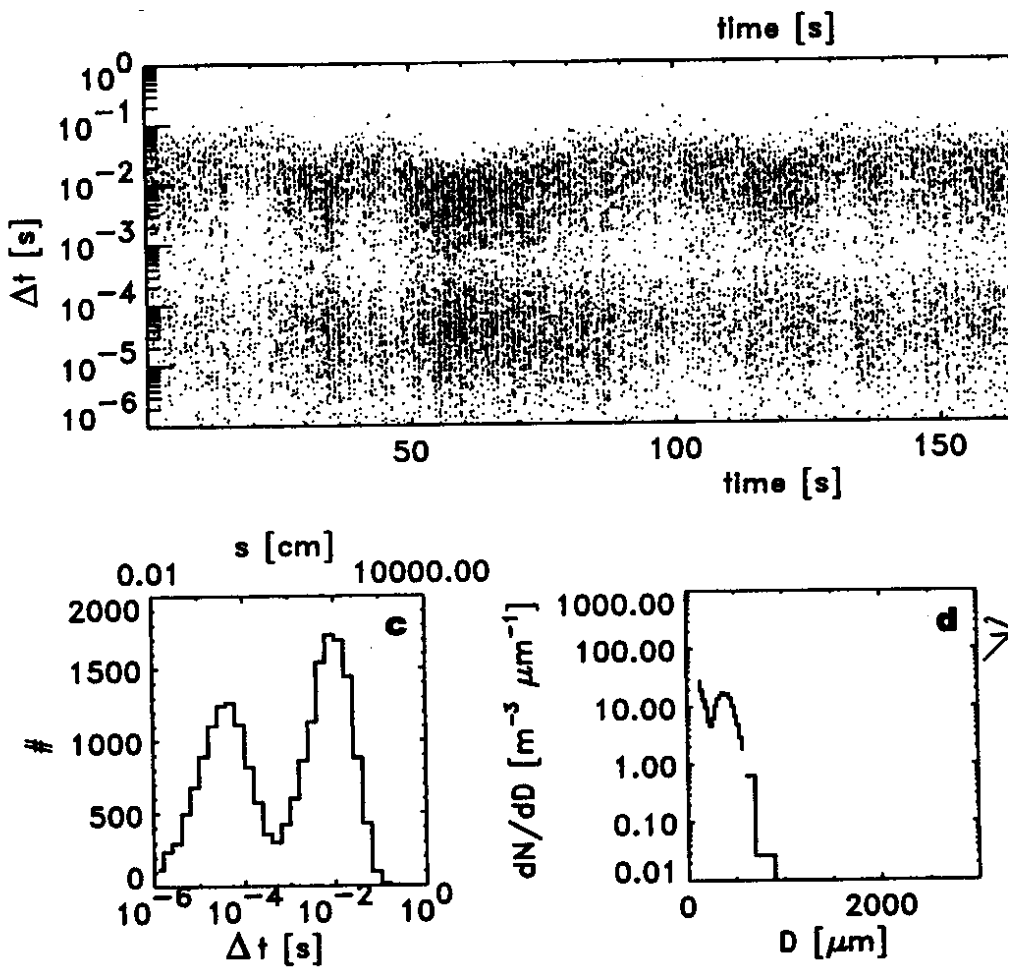
Results: Gamma function fits to observed PSD



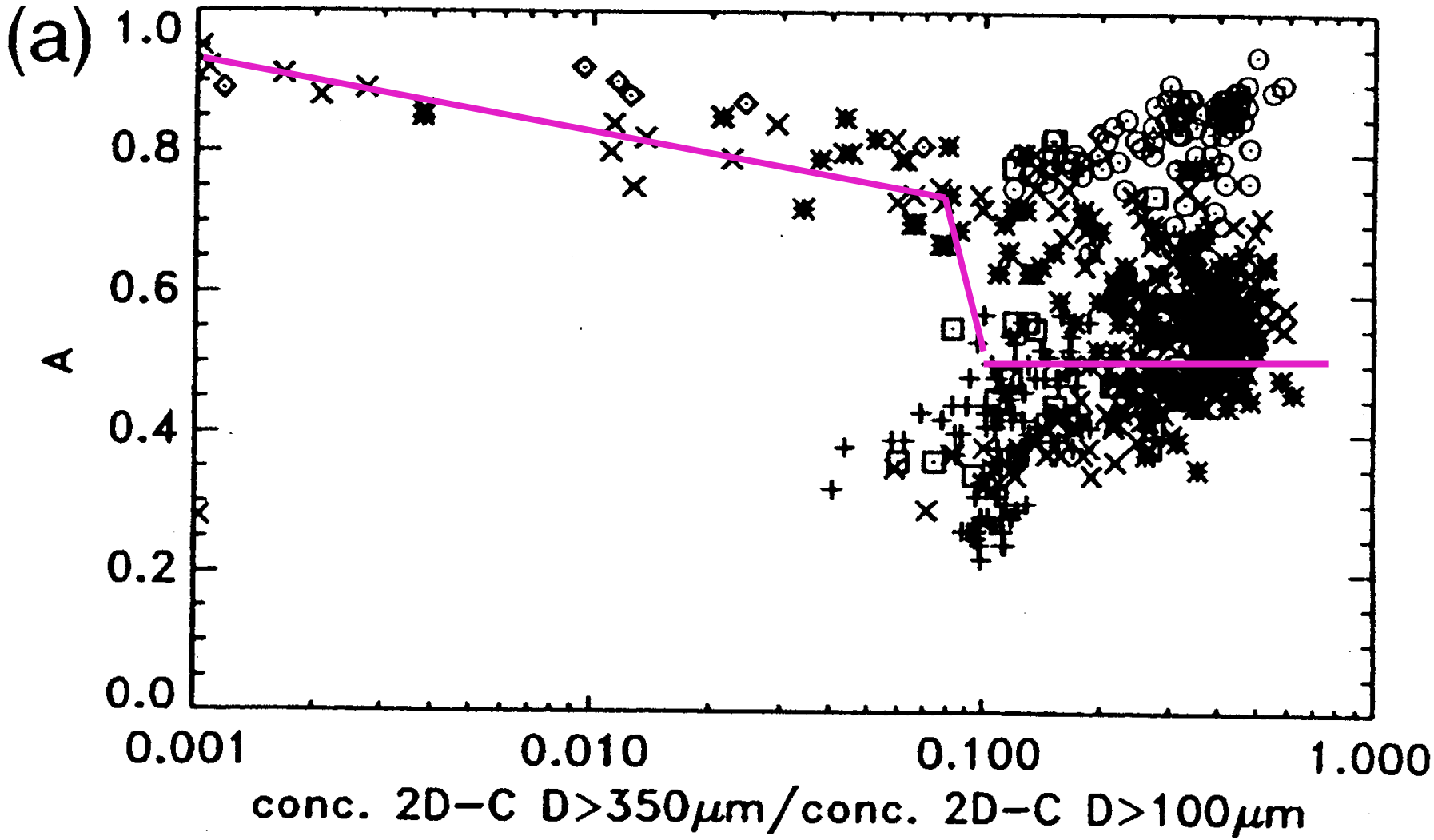
Adjusting the Small Mode to Account for Ice Particle Shattering at FSSP Inlet

- **From Field et al. 2003, J. Atmos. Oceanic Tech.**
- **FSSP interarrival times reveal 2 modes, one mode possibly due to shattering**

J. Atmospheric and Oceanic Technology



A = fraction of the "original" concentration to total concentration of ice particles.
From Field et al. 2003, J. Atmos. Ocean. Tech., 20, 249-261.



Conclusions

1. A method has been developed for fitting two gamma functions to the measured PSD in ice clouds based primarily on the mean, median mass, and median radar reflectivity dimensions of the PSD. The results suggest natural PSD can be described as two populations of particles having different mass-dimension relationships.

2. The methodology provides a way for estimating the ice particle mass-dimension power law relationship that is representative of the large mode.

3. The estimated m-D power or β has a similar magnitude as PSD β estimates in Brown and Francis (1995), Heymsfield (2003) and Heymsfield et al. (2004), where $1.9 \leq \beta \leq 2.4$. The slightly lower values found here may reflect the dependence of ν on the radar reflectivity moment of the PSD.

4. The potential effects from shattering of larger particles can be addressed by attenuating the small mode N_o based on Fig. 10a in Field et al. (2003).

5. This methodology can be used to develop PSD parameterizations for GCMs.