Beer's law, Langley plots, and enhanced absorption in atmospheric windows

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- Universal behavior of transmission for non-grey media
- New methods for approximating transmission in radiative transfer
- Implications for modeled vs. observed absorption in windows

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Beer's Law is harder than it looks

Beer's Law: Atmospheric extinction causes exponential reduction in radiation fields:

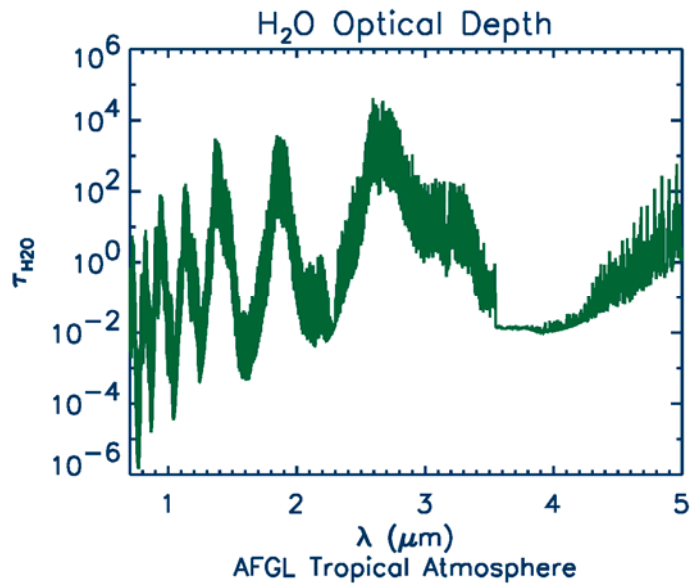
$$I = I_0 \exp[-\tau]$$

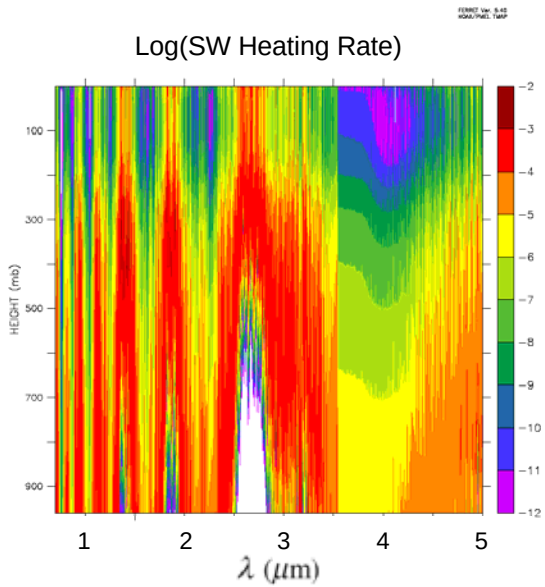
$\tau =$ optical depth

When we want to treat radiation over a range of wavelengths, Beer's law becomes:

$$I = \int I_0 \exp[-\tau_\lambda] d\lambda$$

$\tau_\lambda =$ **spectral optical depth**

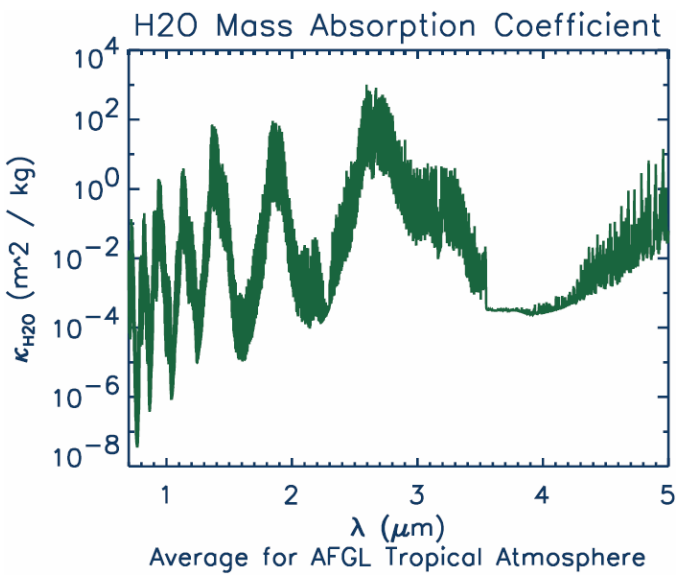


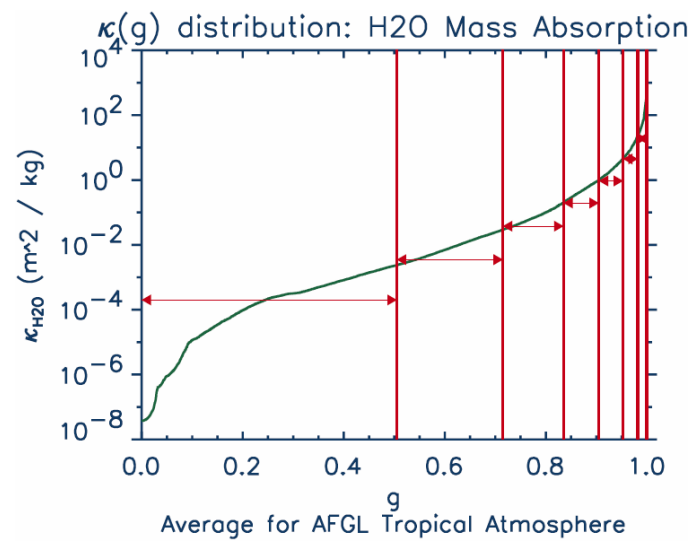


Collins et al, 2006

The solar and infrared spectra exhibit variations in extinction, optical depth, and heating rates of >10 orders of magnitude.

Radiative transfer using k-band theory





$$T(u) = \frac{1}{\lambda_2 - \lambda_1} \int_{\lambda_1}^{\lambda_2} \exp\left[-\kappa(\lambda) u\right] d\lambda$$

$$T(u \quad \hat{T}(u) \simeq \sum_{i=1}^{N} \exp\left[-\hat{\kappa}_i u\right] \delta g_i \quad dg$$

- **In correlated-k codes, the absorption coefficients are sorted by magnitude.**
- The transmission integral should be much easier to approximate in this sorted form.
- **Yet classical approximation methods may not be suitable.**

- **Are there physically and mathematically optimal methods for approximation?**

Methods for representing transmission in AOGCMs

Existing Methods:

- Classical numerical quadrature
- Empirical parameter estimation
- Exponential Sum Fitting of Transmission (ESFT)
- More exotic methods from signal-processing theory

These methods lack:

- Practical error estimates
- Guaranteed accuracy for all paths
- Link between order & accuracy
- Guarantee of global optimality
- Continuous approximations over a range of atmospheric conditions

Our constructive method provides:

- Formal error estimates
- Absolute accuracy for all paths
- Theory for order and accuracy
- Globally optimal solutions
- Mathematical continuity
- New insights into radiative processes

Error bounds on transmission approximation

Let

$\varepsilon =$ chosen limit on error in transmission

$u =$ path length through atmospheric medium

$T(u) =$ exact transmission

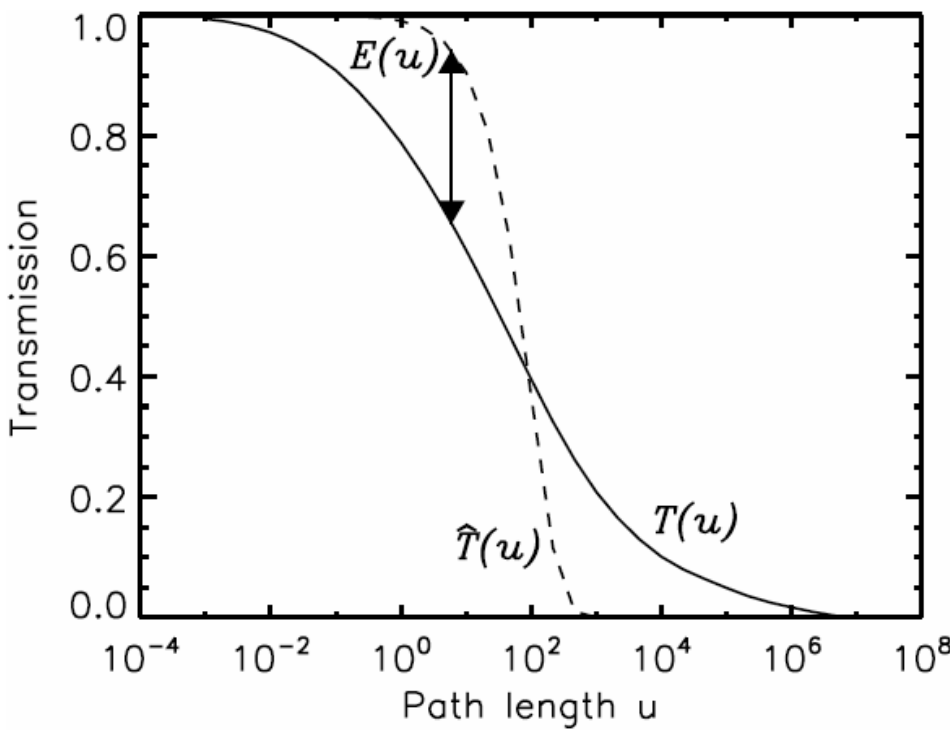
$\hat{T}(u) =$ approximation to transmission

The error in the approximation is

$$E(u) = \left| T(u) - \hat{T}(u) \right|$$

The basic requirement is:

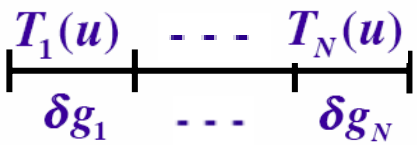
$$\boxed{E(u) \le \varepsilon \qquad \text{for all path lengths } u}$$



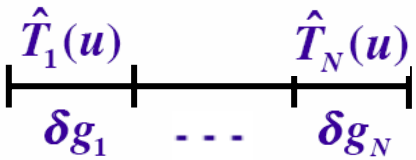
Error bounds on weak and strong transmission

Divide the k-distribution integral into intervals:

$$T(u) = \sum_{i=1}^{N} T_i(u)\delta g_i$$



$$\hat{T}(u) = \sum_{i=1}^{N} \hat{T}_i(u)\delta g_i$$



The error in the approximation for each interval is:

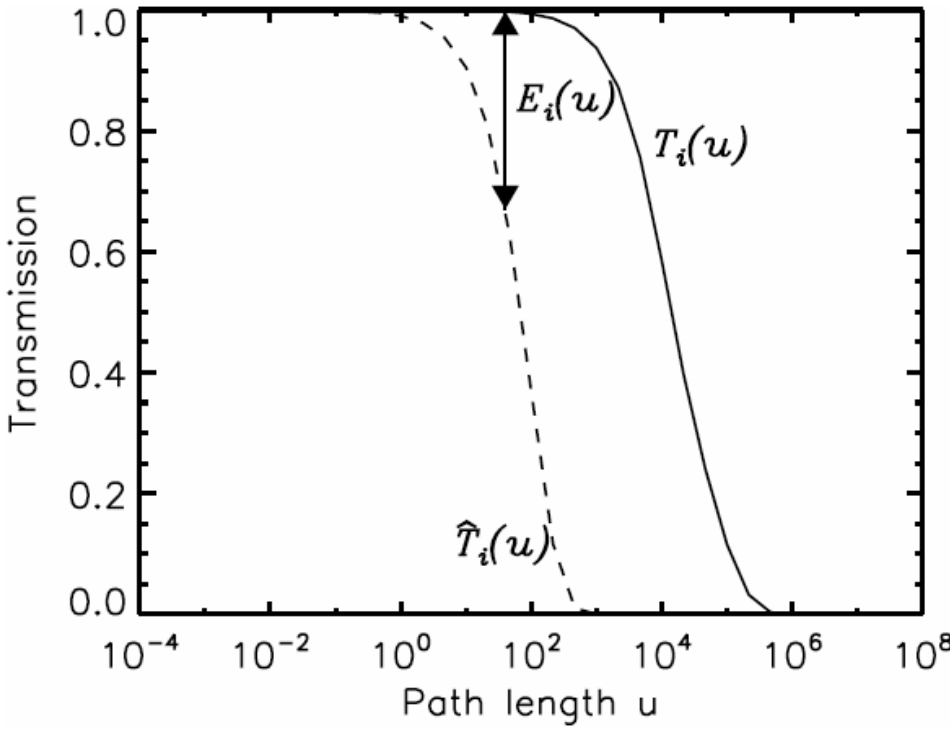
$$E_i(u) = \left| T_i(u) - \hat{T}_i(u) \right|$$

Assume that the error in each interval obeys:

$E_i(u) \le \varepsilon$ for all path lengths u

Then we satisfy our global requirement since

$$\boxed{E(u) \le \sum_{i=1}^{N} E_i(u)\delta g_i = \varepsilon}$$



Variation of transmission with path length

The exact and approximate transmissions are:

$$T_i(u) = \exp[-\kappa_i(u)u]$$

$$\hat{T}_i(u) = \exp[-\hat{\kappa}_i u]$$

where

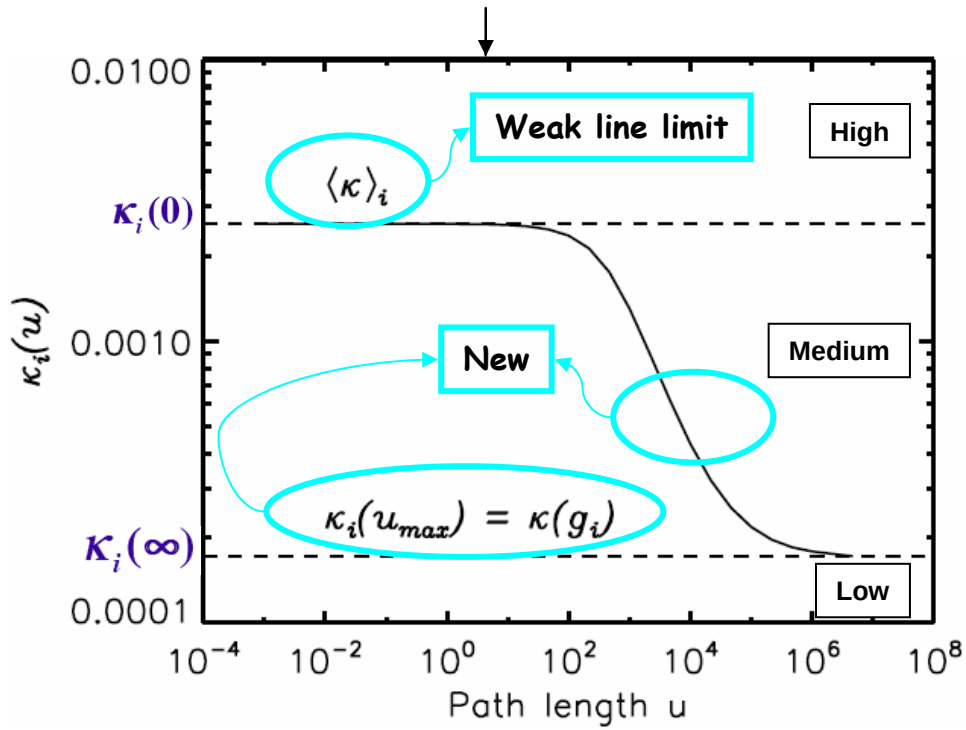
$\kappa_i(u) =$ effective extinction coefficient

$\hat{\kappa}_i =$ approximate extinction coefficient

One can prove for each interval that:

- $\kappa_i(u)$ is a decreasing function of u
- For $u = 0$, $\kappa_i(0)$ is the average extinction
- For $u \to \infty$, $\kappa_i(\infty)$ is the minimum extinction

Optimal choices for $\hat{\kappa}_i$ follow from these properties.



Relationship of specific extinction and error

Low. $\hat{\kappa}_i < \kappa_i(\infty)$

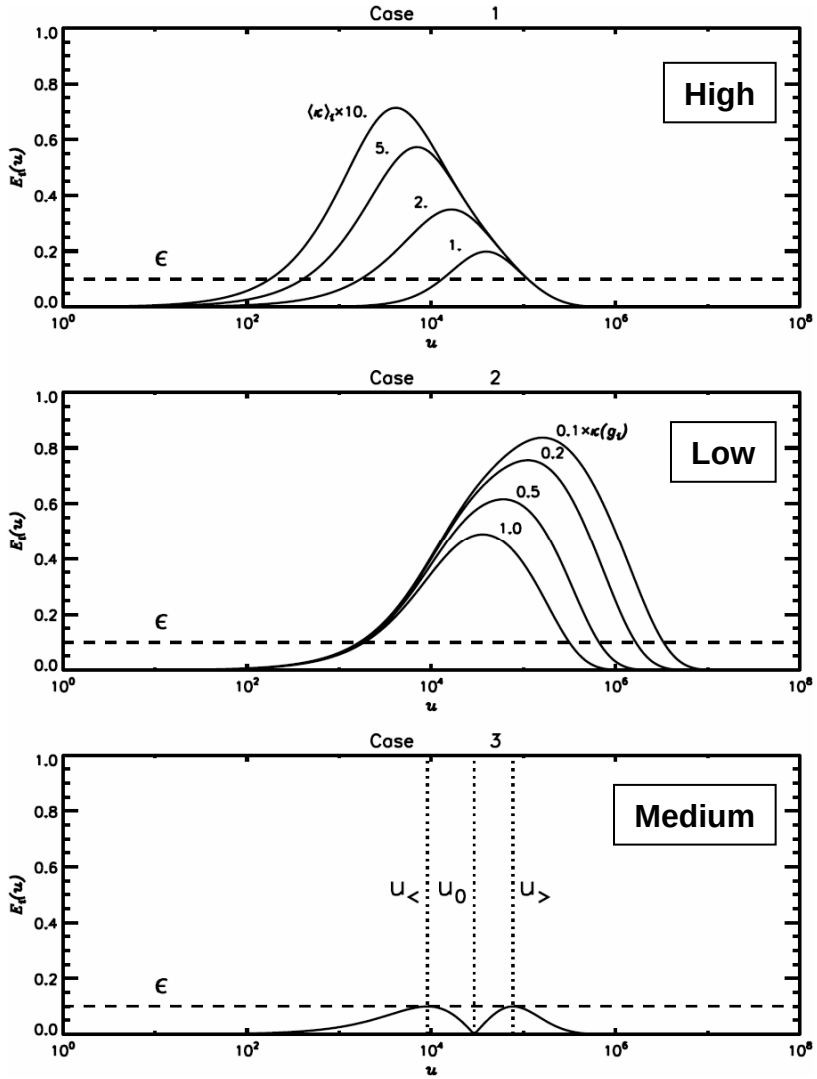
Reject: Errors > Errors for Middle option

Medium. $\kappa_i(\infty) \le \hat{\kappa}_i \le \kappa_i(0)$

Accept

High. $\hat{\kappa}_i > \kappa_i(0)$

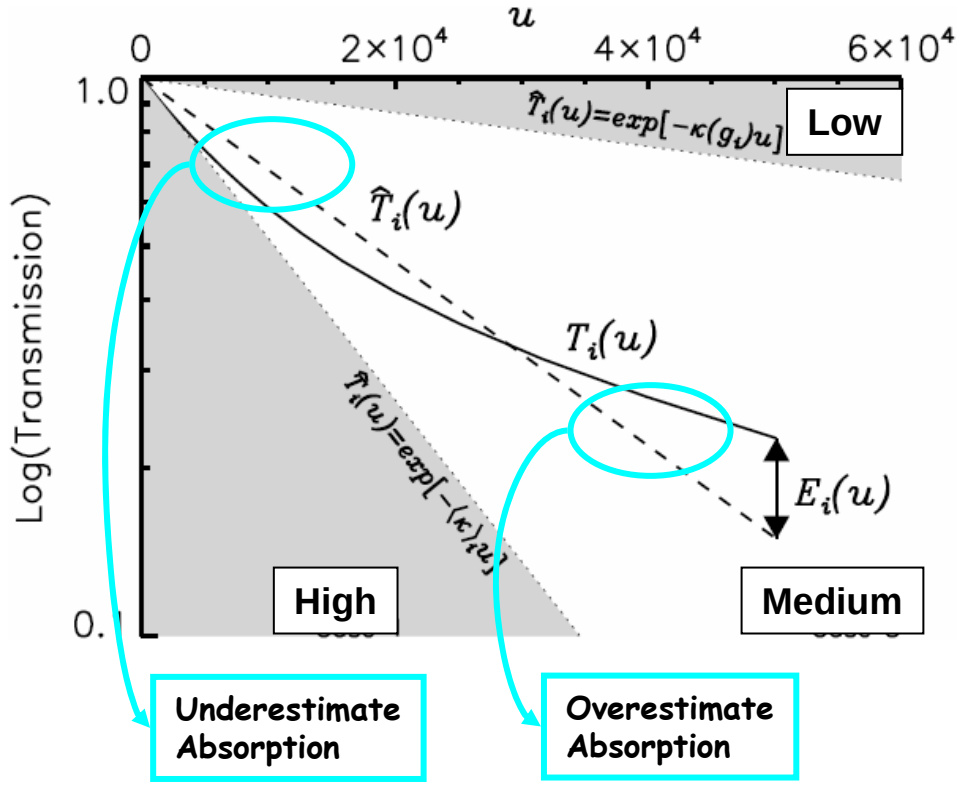
Reject: Errors > Errors for Middle option



Variation of error with path length

For extinctions in the middle range:

- The errors vanish for 3 paths:
  - $u = 0$
  - $u = u_0$ for some $u_0 > 0$
  - $u = \infty$
- The errors are maximized for 2 paths:
  - $u = u_<$ for some $u_< < u_0$
  - $u = u_>$ for some $u_> > u_0$



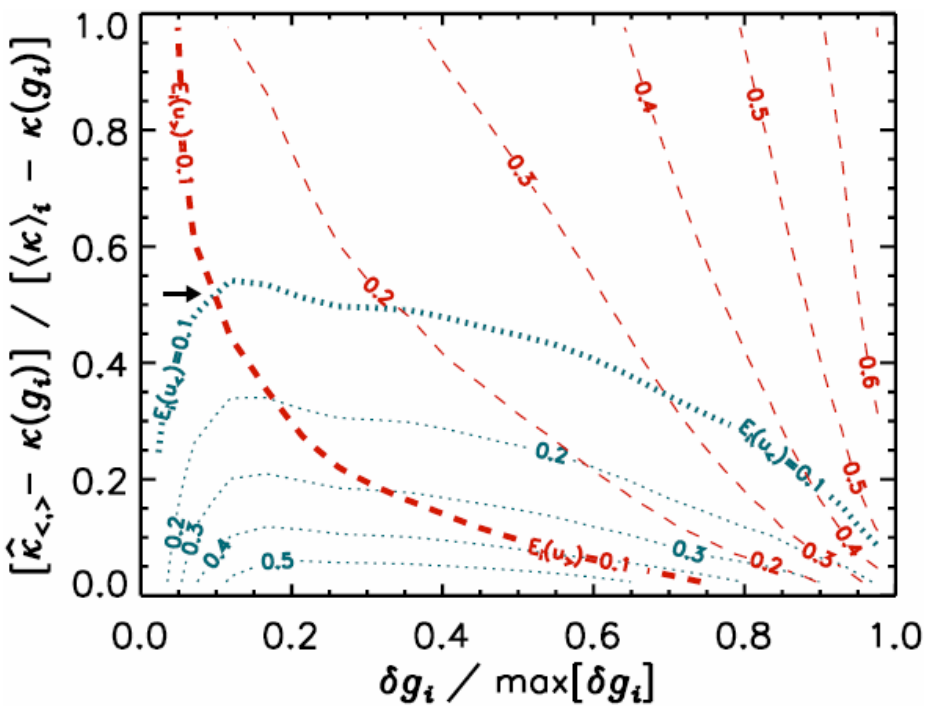
Basic equations for extinction and interval width

Suppose we require the maximal errors obey:

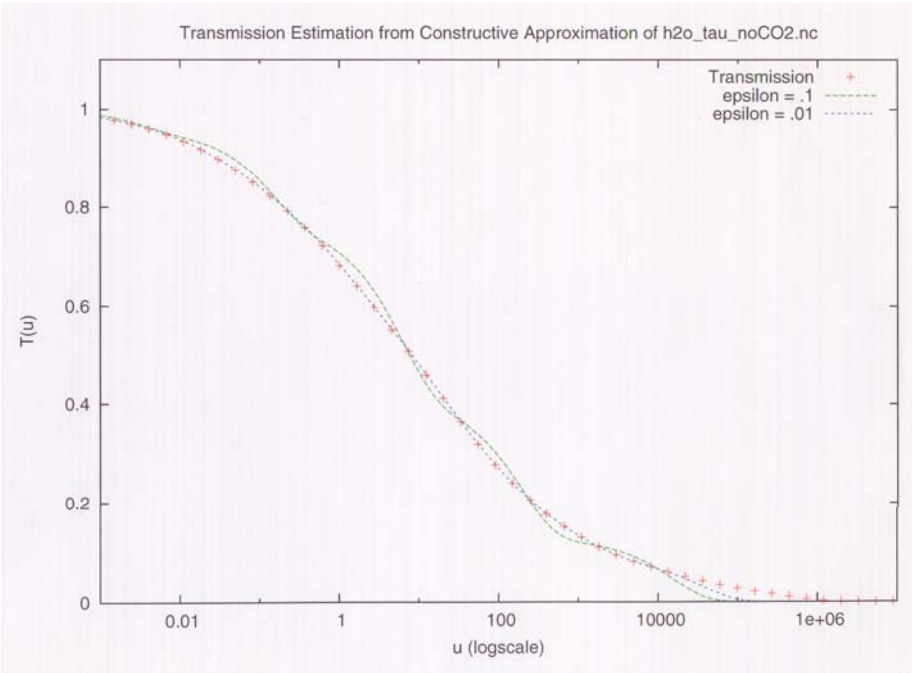
$$E_i(u_<) = \varepsilon$$
$$E_i(u_>) = \varepsilon$$

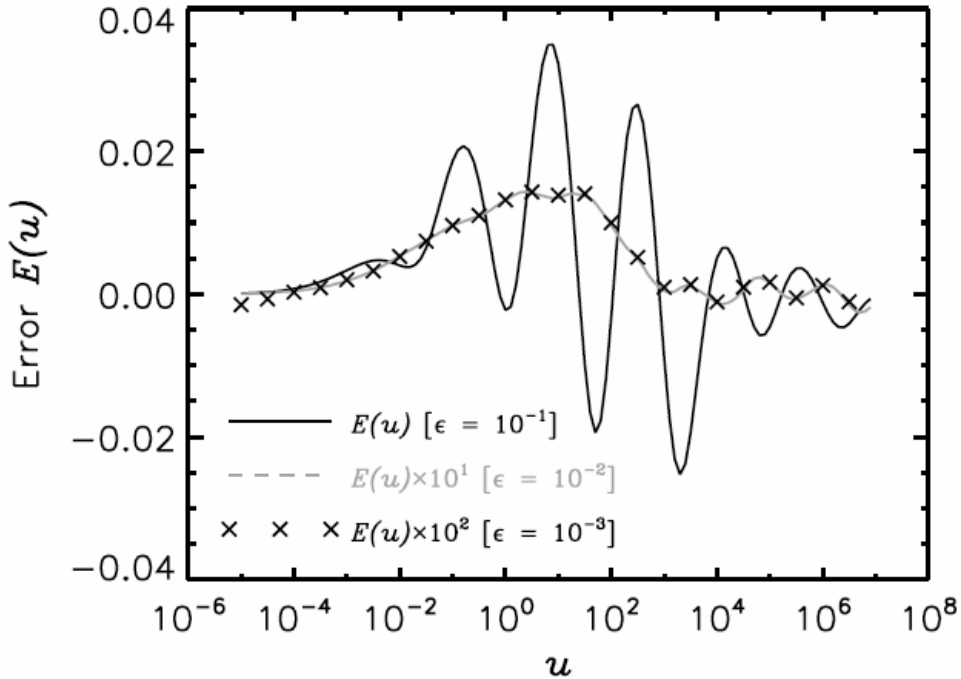
This gives two equations in two unknowns:

$$\left.\begin{aligned} E_i(u_<) &= \varepsilon \\ E_i(u_>) &= \varepsilon \end{aligned}\right\} \Rightarrow \begin{bmatrix} \delta g_i \\ \hat{\kappa}_i \end{bmatrix}$$



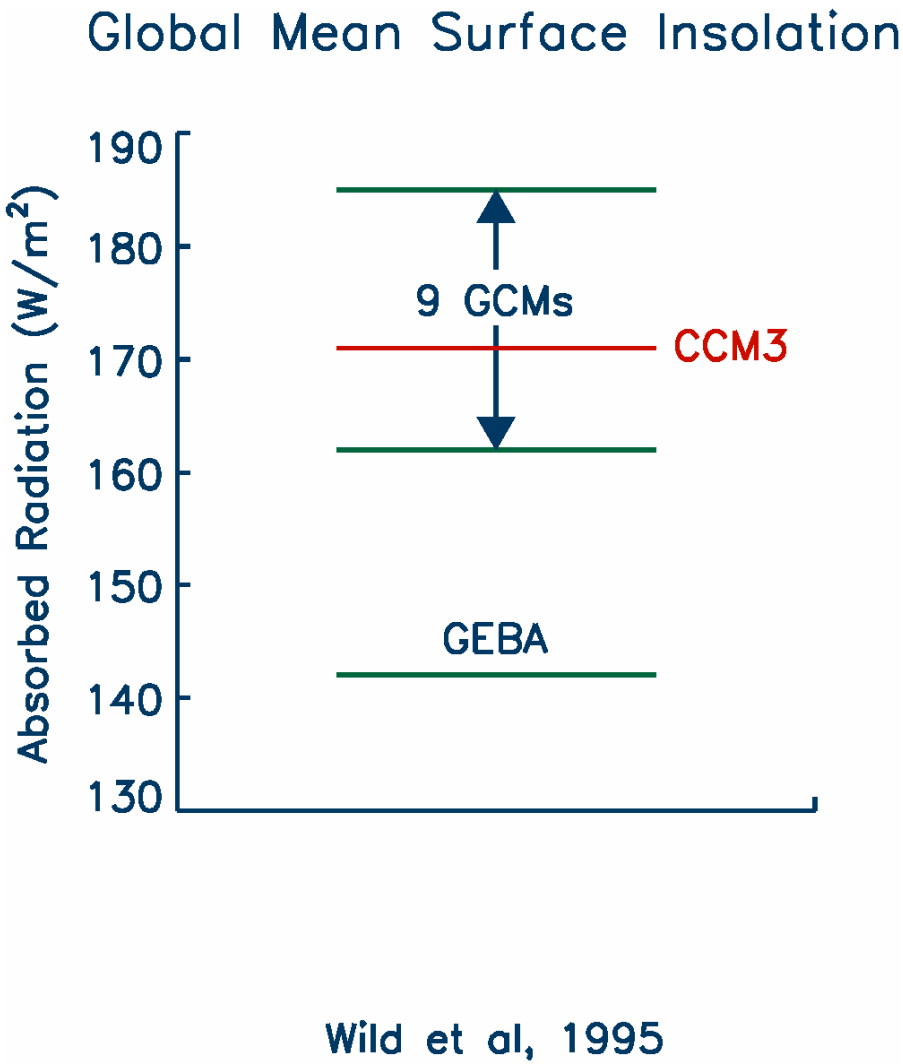
Approximation for Near-IR H_2O Transmission





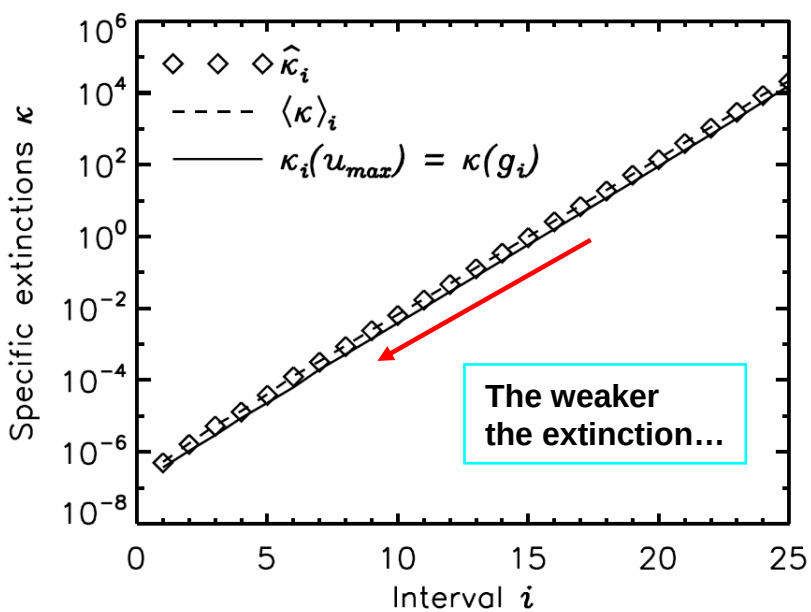
Implications for enhanced shortwave absorption

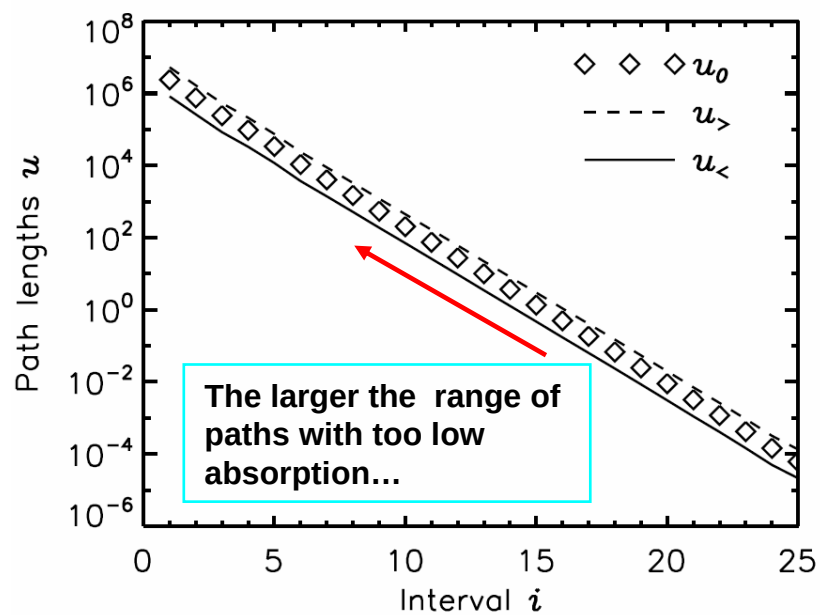
AOGCMs tend to systematically overestimate the surface insolation.



“Anomalous” absorption in atmospheric windows

- Absorption is underestimated for path lengths below a threshold value.
- This underestimation is also typical of other methods for radiative transfer.
- The threshold path length **increases** with **decreasing** absorption.
- **This implies that models systemically underestimate absorption in atmospheric “windows” where extinction is low.**





Summary

- **There are "universal" properties of Beer's law:**
  - The effective extinction is bounded above and below.
  - The effective extinction decreases with increasing path length.
  - Langley plots of non-grey media are convex upward.

- **These properties lead to new transmission approximations:**
  - It is possible to bound the absolute error for <u>any</u> path length
  - The <u>only</u> free parameter is the error bound.
  - The error characteristics are mathematically grounded.

- **Simple approximations <u>must</u> underestimate window absorption.**

- **These methods could lead to new, more robust, radiative transfer.**